# Chapter 2

# Reasoning and Proof

# Section 3
# Deductive Reasoning

# GOAL 1: Using Symbolic Notation

In Lesson 2.1, you learned that a conditional statement has a hypothesis and a conclusion. Conditional statements can be written using symbolic notation, where *p* represents the hypothesis, *q* represents the conclusion, and $\rightarrow$ is read as "implies." Here are some examples:

$$\text{If } \underbrace{\text{the sun is out}}_{p}\text{, then } \underbrace{\text{the weather is good}}_{q}.$$

This conditional statement can be written symbolically as follows:

If p, then q          or          $p \rightarrow q$

To form the converse of an “If p, then q” statement, simply switch p and q.

If $\underbrace{\textbf{the weather is good}}_{q}$, then $\underbrace{\textbf{the sun is out}}_{p}$.

The converse can be written symbolically as follows:

If q, then p        or        $q \rightarrow p$

A biconditional statement can be written using symbolic notation as follows:

If p, then q and if q, then p        or        $p \leftrightarrow q$

Most often a biconditional statement is written in this form:

p if and only if q.

Example 1: Using Symbolic Notation

Let *p* be "the value of x is -5" and let *q* be "the absolute value of x is 5."

a. Write p → q in words.

If the value of x is -5, then the absolute value of x is 5. TRUE

a. Write q → p in words.

If the absolute value of x is 5, then the value of x is -5. FALSE

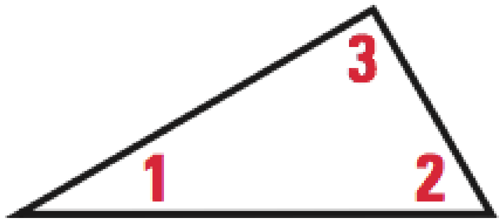
a. Decide whether the biconditional statement p↔ q is true.

FALSE b/c the converse is false.

To write the inverse and contrapositive in symbolic notation, you need to be able to write the negation of a statement symbolically. The symbol for negation (~) is written before the letter. Here are some examples:

| STATEMENT | SYMBOL | NEGATION | SYMBOL |
| --- | --- | --- | --- |
| $\angle 3$ measures 90°. | $p$ | $\angle 3$ does not measure 90°. | $\sim p$ |
| $\angle 3$ is not acute. | $q$ | $\angle 3$ is acute. | $\sim q$ |

The inverse and contrapositive of p → q are as follows:




**Inverse:** ~p → ~q

If <3 does not measure 90°, then <3 is acute.

**Contrapositive:** ~q → ~p

If <3 is acute, then <3 does not measure 90°.

Notice that the inverse is false, but the contrapositive is true.

Example 2: Writing an Inverse and a Contrapositive

Let *p* be “it is raining” and let *q* be “the soccer game is canceled.”

a. Write the contrapositive of $p \rightarrow q$. $(\sim q \rightarrow \sim p)$

If the soccer game is not canceled, then it is not raining.

a. Write the inverse of $p \rightarrow q$. $(\sim p \rightarrow \sim q)$

If it is not raining, then the soccer game is not canceled.

Recall from Lesson 2.1 that a conditional statement is equivalent to its contrapositive and that the converse and inverse are equivalent.

| Equivalent Statements |
|---|
| **Conditional Statement**<br>$p \rightarrow q$<br>If the car will start,<br>then the battery is charged. |
| **Contrapositive**<br>$\sim q \rightarrow \sim p$<br>If the battery is not charged,<br>then the car will not start. |

| Equivalent Statements |
|---|
| **Converse**<br>$q \rightarrow p$<br>If the battery is charged,<br>then the car will start. |
| **Inverse**<br>$\sim p \rightarrow \sim q$<br>If the car will not start,<br>then the battery is not charged. |

In the table above the conditional statement and its contrapositive are true. The converse and inverse are false.

GOAL 2: Using the Laws of Logic

Deductive reasoning uses facts, definitions, and accepted properties in a logical order to write a ___logical argument___. This differs from inductive reasoning, in which previous examples and patterns are used to form a conjecture.

Example 3: Using Inductive and Deductive Reasoning

The following examples show how inductive and deductive reasoning differ.

a. Andrea knows that Robin is a sophomore and Todd is a junior. All the other juniors that Andrew knows are older than Robin. Therefore, Andrea reasons inductively that Todd is older than Robin based on past observations.

b. Andrea knows that Todd is older than Chan. She also knows that Chan is older than Robin. Andrea reasons deductively that Todd is older than Robin based on accepted statements.

There are two laws of deductive reasoning.

The first is the ____law of detachment____, shown below.

The ___law of syllogism___ follows in the next slides.

**LAW OF DETACHMENT**

If $p \rightarrow q$ is a true conditional statement and $p$ is true, then $q$ is true.

Example 4: Using the Law of Detachment

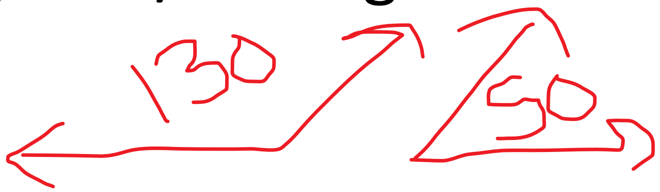
State whether the argument is valid.

a. Jamal knows that if he misses the practice the day before a game, then he will not be a starting player in the game. Jamal misses practice on Tuesday so he concludes that he will not be able to start in the game on Wednesday.

Valid → p → q is true, p is true → q is also true

a. If two angles form a linear pair, then they are supplementary; <A and <B are supplementary. So, <A and <B form a linear pair.

Not valid → just b/c 2 angles are supplementary, doesn’t mean they're a linear pair



## LAW OF SYLLOGISM

If $p \to \cancel{q}$ and $\cancel{q} \to r$ are true conditional statements, then $p \to r$ is true.

Example 5: Using the Law of Syllogism

**ending of 1 has to match the beginning of the other

Zoology: Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a bird is the fastest bird on land, then it is the largest of all birds.
2. If a bird is the largest of all birds, then it is an ostrich.
3. If a bird is a bee hummingbird, then it is the smallest of all birds.
4. If a bird is the largest of all birds, then it is flightless.
5. If a bird is the smallest bird, then it has a nest the size of a walnut half-shell.

1&2 → If a bird is the fastest bird on land, then it is an ostrich.

3&5 → If a bird is a bee hummingbird, then it has nest the size of….

1&4 → If a bird is the fastest bird on land, then it is flightless.

# Example 6: Using the Laws of Deductive Reasoning

Over the summer, Mike visited Alabama. Given the following true statements, can you conclude that Mike visited the Civil Rights Memorial?

If Mike visits Alabama, then he will spend a day in Montgomery.

If Mike spends a day in Montgomery, then he will visit the Civil Rights Memorial.

If Mike visits Alabama, then he will visit the Civil Rights Memorial.

L.O.D. → p→q true, p is true → q is true

→ he did visit the C.R.M.